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Realized Volatility and India VIX

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I. Introduction

While the market return of a stock is difficult to predict, there are well established models to predict return volatility. It has been observed in early sixties of the last century (Mandelbrot 1963) that stock market volatility exhibits clustering, where periods of large returns are followed by periods of small returns .Later popular models of volatility clustering were developed by Engle (1982) and Bollerslev (1986). The autoregressive conditional heteroskedastic (ARCH) models (Engle, 1982) and generalized ARCH (GARCH) models (Bollerslev, 1986) have been extensively used in capturing volatility clustering in financial time series (Bollerslev et al. 1992). Using data on developed market, several empirical studies (Akgiray, 1989; West et al, 1993) have confirmed the superiority of GARCH-type models in volatility predictions over models such as the naïve historical average, moving average and exponentially weighted moving average (EWMA). GARCH models can replicate the fat tails observed in many high frequency financial asset return series, where large changes occur more often than a normal distribution would imply. Financial markets also demonstrate that volatility is higher in a falling market than it is in a rising market. This asymmetry or leverage effect was first documented by Black (1976) and Christie (1982). Empirical results also show that augmenting GARCH models with information like market volume or number of trades may lead to modest improvement in forecasting volatility (Brooks, 1998; Jones et al, 1994).

It is an empirical question to establish whether conditional volatility models better capture the underlying volatility of the asset return. A model free estimation of implied market volatility, VIX (volatility index), was introduced by CBOE in 1993 based on S&P 100 options. It was believed that VIX would be very close to realized volatility. In September 2003 the Chicago Board Options Exchange (CBOE) modified the methodology for VIX so that (1) the new VIX is now based on prices of S&P 500 (rather than S&P 100) options, and (2) the new VIX formula takes into account a broader range of strike prices (rather than using only near-the-money strikes as the original-formula index did). Each strike price is weighted, with at-the-money strikes having the most weight. The new formula is intended to make VIX a better index for investors who manage risks associated with the growing markets for volatility and variance swaps. A volatility swap is a forward contract on realized historical volatility of the underlying equity index. In such a contract, the buyer receives a payout from the counterparty selling the swap if the volatility of the stock index realized over the life of swap contract exceeds the implied volatility swap rate noted at the inception of the contract. The implied volatility is the fixed “swap rate”, and is established by the writer of the swap at the inception of the contract. VIX can be used as implied volatility input in this context.

The old VIX of CBOE was a proxy for at-the-money implied volatility and the new VIX is a proxy for variance swap rate. The CBOE volatility index measures the implied volatility of S&P 500 index options at a 30 day time horizon. VIX generally measures fear or complacency in the market. In March 2004 the first-ever U.S. exchange trading of volatility-based contracts began as VIX futures were launched on the new CBOE Futures Exchange (CFE). It was observed that the VIX Index has tended to sharply increase when the S&P 500 fell rapidly. So, traders found it profitable to use VIX futures for hedging index options positions.

Indian version of VIX (called India VIX) was introduced by the National Stock Exchange (NSE) in November 2007. The methodology of India VIX is based on VIX of CBOE. India VIX is based on Nifty 50 index options contracts. The method does not use any option pricing model, but simply uses near and mid-month options bid and offer prices to derive the implied volatility. India VIX is expressed an annualized percentage of volatility for next 30 days. Of course, there has not yet been any product launched on VIX in India.

This paper attempts to compare the performance of conditional volatility model (GARCH) and VIX in predicting underlying volatility of the Nifty 50 index. The underlying volatility of Nifty 50 index is captured using high frequency data. Several approaches (e.g., Corsi et al.,2001, Andersen et al. 2003, Bandi and Russell ,2004, and Zhang et al. 2005) to estimate realized volatility are considered . The performance of VIX and GARCH models are evaluated using diagnostics, like *mean absolute error (MAE), root mean squared error (RMSE)*.

The remaining section of the paper continues as follows. Section II describes the methodology and data. Empirical results are reported and discussed in section III. Finally, concluding remarks are made in section IV.

II. Data and Methodology

Let r_{ti} , $i = 1 \ldots . n_t$ denote log of price relatives at an intraday time-point *i* on day *t*, where n_t is the number of return observations obtained by using prices *n* times per day[1]. Then daily return on day *t* is calculated as $r_t = \sum_{i=1}^{n_t} r_{ti}$

Generally, the conditional mean μ_t of such return series $\{r_t\}$ can be modelled using a simple time series model such as a stationary ARMA(*p,q*) model, i.e.,

$$r_{t=\mu_t+\varepsilon_t}, \quad \mu_t = \emptyset_0 + \sum_{i=1}^{p} \emptyset_i \, r_{t-i} - \sum_{j=1}^{q} \theta_j \, \alpha_{t-j,} \qquad (1)$$

$$\varepsilon_{t=z_t\sqrt{h_t}}$$

where the shock (or mean-corrected return) ε_t represents the shock or unpredictable return, and *p*, *q* are non-negative integers. Z_t is a white-noise with mean zero and variances one and h_t is the conditional variance of ε_t. The conditional variance, then, can be modeled in a GARCH (p,q) process as:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, h_{t-j} \qquad (2)$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$,

$\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$, with $\alpha_i = 0$ for *i>p* and $\beta_j = 0$ for *j >q*,

We have fitted simple GARCH (1, 1) model initially for conditional volatility

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad (3)$$

[1] High frequency data obtained from NSE (National Stock exchange)

The use of higher order GARCH model is deliberately avoided as there are enough empirical evidence that a simple GARCH (1,1) is found to adequately fit many financial time series (Sharma et al., 1996).

Our return model was represented as , $\mu_t = constant + \varepsilon_t$ [2]. In our procedure, we used daily data for the period January 2005-Aug 2007 to estimate the initial Garch (1,1) model and predict for first out-of-sample day in September 2007. Thereafter, we kept on including the day from the out-of-sample period to the in-sample period to estimate the Garch (1,1) model and predict for the next day. For example to predict for 19th Oct 2007, we used data from Jan 2005-18th Oct 2007 to estimate the Garch(1,1) model and then predict for 19th Oct, 2007. The Garch(1,1) model was developed in C++ using the newmat 10 code library (Davies 2006).

For VIX construction, we followed the VIX methodology as adopted by NSE (NSE 2007) and originally developed by Whaley (1993). Our out-of-sample test data includes period from September 2007 to November 2008. VIX was introduced in India in November 2007 and since then the data is available from the NSE website. So effectively, we needed to compute VIX only for September-October 2007. The methodology is detailed below for the sake of completeness.

VIX computation makes use of two nearest months' Nifty call and put options contracts' data to bracket the 30 day period. It makes use of the best bid and best ask price data for each Nifty call and put option contract in the two nearest months to compute VIX for that day. In our study, we considered all available Nifty call and put contracts for each day. This data can be freely obtained from the NSE website. Next we had derivatives' orders snapshot data available for a maximum of five times in a day i.e. at 11 am, 12 pm, 1pm, 2 pm and 3 pm. This data was purchased from NSE. Using the orders in these snapshot files, we computed the best bid and best ask price for each Nifty call and put option contract. We noticed that in addition to bid prices being zero for many option

[2] We initially used Nifty daily return series for the period January 2005-August 2007 and checked that the return model and GARCH (1,1) used here, is a good fit for Nifty return series. The parameter estimates of the GARCH model are not reproduced for brevity.

contracts, many times, ask prices were also zero. We then followed the step by step methodology followed by NSE, to compute VIX during period September-October 2007, with the additional condition that in the procedure wherever option with zero bid prices were ignored we also checked for zero ask prices and ignored such contracts. Similarly, the stopping condition while selecting contracts dictates stopping when two consecutive contracts with zero bid prices are encountered. We added the condition that we ignore any further contracts, even when two successive contracts with zero ask prices as well, are encountered. The reasoning being that zero bid or ask indicates no trade. The risk free interest rate used for computation was 7.1858% (91-day T-Bill rate)

Realized Volatility

It is true that unlike prices, volatilities are not directly observable in the market, and it can only be estimated in the context of a model. However, Merton (1980), Andersen et al (2001) observed that by sampling intraday returns sufficiently frequently, the realized volatility (measured by simply summing intraday squared returns) can be treated as the observed volatility. This observation has profound implication for financial markets (Brooks, 1998) in that (a) the realized volatility provides a better measure of total risk (value at risk) of financial assets, and (b) it can lead to better pricing of various traded options.

Ideally, the average daily returns variance can be estimated more accurately by summing all the squared intraday available returns, rather than calculating the squared daily return (McAleer and Medeiros, 2008).

$$RV_t = \sum_{i=0}^{n_t} r_{t,i}^2 \qquad (4)$$

Andersen et al. (2003) showed that the realized variance using all data available is a consistent estimator of the integrated variance when there is no microstructure noise. The integrated variance is considered as the measure of “true” daily volatility. The problems with using all available tick data are well known:

(a) Prices are observed at discrete and irregularly spaced intervals. Hence, use of all data would imply returns with uneven time intervals.

(b) Tick-by-tick data would have microstructure noise (e.g., bid-ask bounce, execution of staggered order) (Bandi and Russell, 2008) which leads to divergence in observed price process and true price process. The noise causes a strong negative first –order autocorrelation in tick-by-tick returns (Zhou, 1996)

On the other hand, if too few observations are used to estimate daily volatility, that would not be representative of integrated variance. Hence, realized volatility estimate is a trade-off between capturing all underlying information and avoiding market microstructure effects. Use of evenly-spaced high frequency returns would necessarily require some form of interpolation among prices recorded around the endpoints of given sampling intervals. There are many ways in which one can sample the data (McAleer and Medeiros, 2008) - *calendar time sampling, transaction time sampling, business time sampling,* and *tick time sampling.* The most popular sampling method is *calendar time* sampling where artificial equidistant calendar time intervals are used. In transaction time sampling, prices are recorded every *m*th transaction (instead of every *m*th minute). In business time sampling prices are sampled on a business time scale defined by the cumulative number of transactions as opposed to a calendar time scale (e.g., price after every *n*th trade). Lastly, prices are recorded at every price change in tick time sampling. This paper uses calendar time sampling although there are evidences that realized variance measures using business time sampling performed better than calendar time sampling (Oomen, 2004)

Andersen et al, (2000) started with a thirty-minute time interval to measure realized volatility. But Corsi et al (2001) observed , using foreign exchange data, that if one uses even higher frequency data, the microstructure effect is incoherent price formation which leads to a strong negative first-order autocorrelation for tick-by-tick returns. The same study found significant first-order positive autocorrelation for stock returns at higher frequency. Therefore, Corsi et al (2001) suggests that one can remove the volatility bias by filtering the return series with a AR(1) or MA(1) process and a (filtered) realized volatility may be estimated using the filtered series. However, almost at the same time, Andersen et al. (2001, 2003) proposed that one can safely use a sampling frequency of 5-minute returns without having to bother about autocorrelation effects, and hence the filtering need. But the issue of optimal sampling frequency is still wide open.

In our computational study, we used 5 min daily return series sampled over November 2006 to November 2008 to initially fit a MA(1) model and then used the residual of this series to compute realized volatility as

$$RV_i = (1-\hat{\theta})^2 \sum_{j=1}^{m} \epsilon_{ij} \;, \qquad (5)$$

where *m* is the number of 5 min returns' observations on day *i.*

In our case, $\hat{\theta} = 0.00769$

There are several studies on optimal sampling frequency. An earlier paper by Bandi and Russell (2004) proposed an approximation for optimal sampling frequency. The approximate optimal sampling frequency is chosen as the value

$$\delta_i^* = \frac{1}{M_i^*} \qquad (6)$$

where,

$$M_i^* = \left(\frac{\widehat{\bar{Q}}_i}{\widehat{\propto}}\right)^{1/3}$$

$$\widehat{\propto} = \left(\frac{1}{nM^{high}} \sum_{i=1}^{n} \sum_{j=1}^{M^{high}} \check{r}_{j,i}^2\right)^2 \text{, M is highest frequency}$$

$$\widehat{Q_i} = \frac{M^{low}}{3} \sum_{j=1}^{M^{low}} \hat{r}_{j,i}^4 \text{ , } M^{low} \text{ is low frequency}$$

Then realized volatility is estimated based on M_i^* as below:

$$RV_i^{(M_i^*)} = \sum_{j=1}^{M_i^*} \tilde{r}_{j,i}^2 \qquad (7)$$

In our computational study, we used data from January 2007 till the day for which the optimal sampling frequency is to be estimated. Our highest frequency is 1 min data and lowest 15 min. Our normal day contains minute by minute return for the time period 9:56 am – 3:30 pm i.e. 335 min.

Since we have minute data available, if the optimal time came out in fraction, it was rounded to the nearest minute.

However, Zhang et al. (2005) showed that the sparse sampling method is not an adequate solution to the problem and proposed a subsampling method in order to estimate integrated variance consistently even in the presence of microstructure noise. Such an estimator is termed as the Two Time Scales Estimator (TTSE). Zhang et al. (2005) pointed out that once an optimal sampling frequency is estimated based on minimization of mean squared error (MSE) using regularly or irregularly spaced data, the full grid of data (e.g., all data in a day) is to be partitioned into K non-overlapping subgrids. Suppose, the full grid, $\Delta_t = \{\tau_0, \ldots \tau_{nt}\}$, is partitioned into K non-overlapping subgrids, $\Delta_t^{(k)}, k = 1, \ldots K$, such that:

$\Delta_t = \cup_{k=1}^{K} \Delta_t^{(k)}$, where $\Delta_t^{(k)} \cap \Delta_t^{j} = \emptyset, when\ k \neq j$

If $n_t^{(k)}$ is the number of observations in each subgrid, one can define realized volatility (RV) for each subgrid k as:

$$RV_t^{(k)} = \sum_{i=1}^{n_t^{(k)}} r_{t,i}^2 \qquad (8)$$

Then the daily RV is estimated using all subgrids and all available data as (Zhang et al., 2005):

$$RV_t^{ZMA} = \frac{1}{K}\sum_{k=1}^{K} RV_t^{(k)} - \frac{\overline{n_t}}{n_t} RV_t^{(all)} \qquad (9)$$

Where n_t is the number of observations in the full grid, and

$$\bar{n}_t = \frac{1}{K}\sum_{k=1}^{K} n_t^{(k)} = \frac{(n_t - K + 1)}{K}$$

We have used the optimal sampling frequency, as estimated using Bandi and Russell(2004), as the sampling time. In order to avoid a situation of using all the datapoints, in case the optimal sampling frequency came out to be 1 minute, we used 2 minute as our sampling interval.

This study uses all the variants of RV measures- (a) Corsi et al.(2001), (b) Andersen et al. (2003), (c) Bandi and Russell (2004), and (d) Zhang et al. (2005).

III. Results and Analysis

In this section, we give the results of our computational study. We used the period from September 2007 to November 2008 as our out-of-sample period. We computed/obtained values for VIX for the period. We used VIX for day t-1 as an indicator for volatility for day t. We estimated Garch(1,1) model using data up to day t –1 and used it for predicting volatility for day t . We then computed the realized volatility for day t using the four methods described above. In our study, for comparison we considered only those days where we were able to compute all the volatility numbers. It is worthwhile to note that while computing VIX during period Sept-Oct 2007, we were not able to compute VIX for a few days due to lack of data. Again while computing realized volatility using intraday data, we computed volatility only for the normal days i.e. the days for which we have all the minute by minute return data available between 9:56am-3:30 pm. We used data up to 3:30 pm only for each day, to maintain uniformity in our daily intraday return series.

Figure 1 shows volatility for different out-of-sample days. As it can be observed GARCH volatility seems to have an upward bias on many days. However on days of extreme volatility it seems to be closer to the realized volatility than the VIX.

Table 1, gives the RMSE and MAE computed for VIX and GARCH with different realized volatilities. As observed, VIX appears to be a better predictor of actual volatility than GARCH(1,1) model.

Figure 1: VIX, GARCH and Realized Volatility

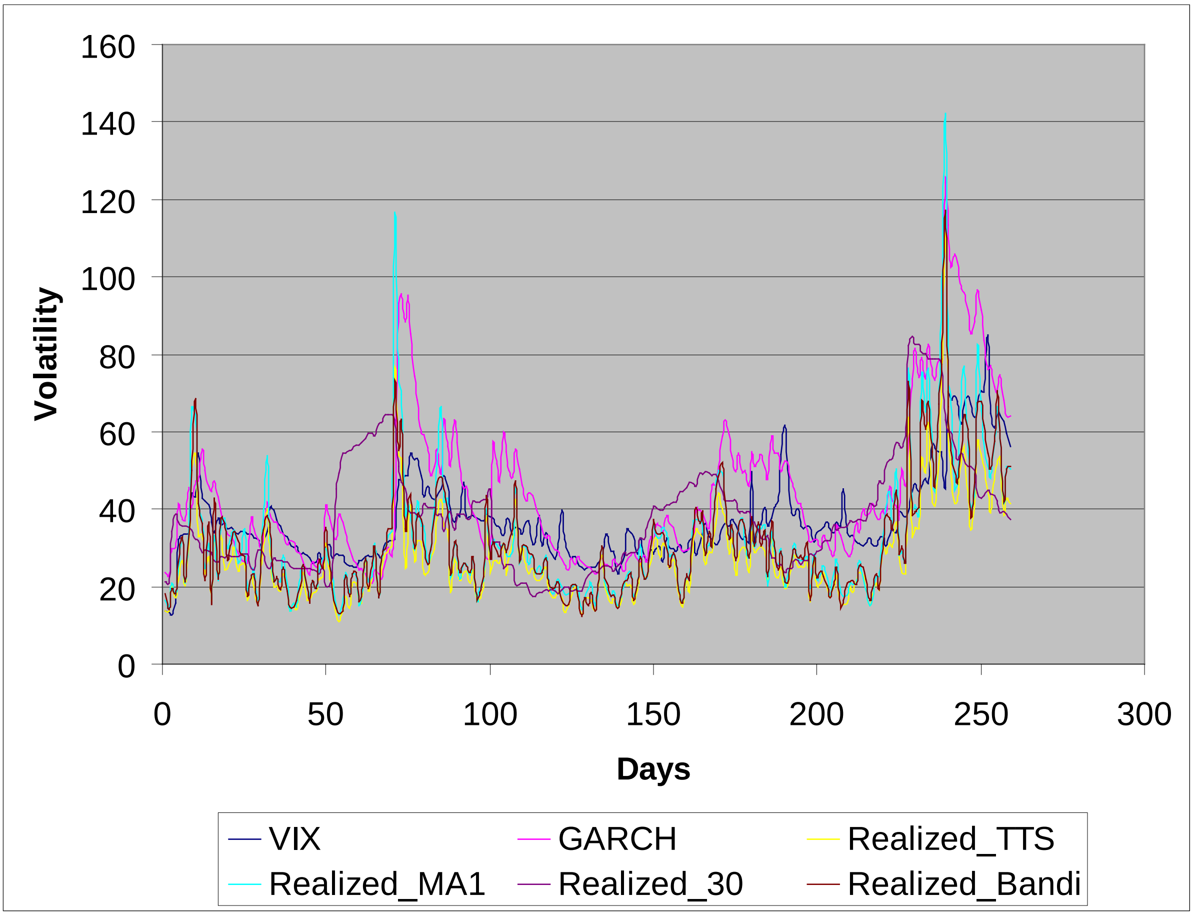

Table 1: Volatility errors

Realized Volatility	RMSE VIX	RMSE GARCH	MAE VIX	MAE GARCH
MA(1) Residuals	14.57	18.49	10.46	14.50
Bandi and Russell (2004)	12.65	18.33	9.84	14.45
TTSE	14.13	21.50	11.61	17.46
30 Calendar days	15.37	20.03	12.10	14.94
Andersen et al.	14.58	18.35	10.39	14.33

IV. **Conclusions**

Previous work (e.g., Day and Lewis, 1992, Lamoureux and Lastrapes, 1993, and Canina and Figlewski, 1993) on implied volatility indicates that implied volatility fails to predict ex-post realized volatility. Also as the implied volatility as estimated from options prices is model-based; it suffers from measurement error. Another difficulty in using Black-Scholes (1973) option pricing model in estimating implied volatility is that the model cannot be used to price index options because of prohibitive transaction costs associated with hedging of options in the cash index market (Christensen and Prabhala, 1998). In this paper we have used VIX as implied volatility estimator. VIX has two advantages- (a) it is a model free estimate and hence free from model error; and (b) it can be conveniently used to measure implied volatility of the stock as well as the index. The present study uses it to measure volatility of the index.

The present study also attempts to measure realized volatility using several estimators available in the literature. The main finding of the study is that forecast error is minimum for VIX. This indicates that a model-free estimator of volatility captures underlying volatility better than an econometric model of volatility (GARCH). This has significant implication for a volatility trader- she can observe the VIX, as disclosed by NSE, and have a view on the volatility of the underlying cash market.

The study could be further extended to compare similar model-free estimate of stock volatility with its realized volatility.

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