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Decision making framework for investments in oil industry: An application of Real Options

by

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Introduction

Uncertainty is one of the major challenges in any business enterprise. There is uncertainty about prices, uncertainty about supply and demand, uncertainty about trade and even uncertainty about political environments. Uncertainty however is not all bad. It creates certain opportunities which when exploited appropriately can create value for the company. As such it is essential that businesses realize the decision flexibilities they have and how they can put a value to the flexibility as to enable comparisons.

These uncertainties create opportunities to capture future value depending on a particular view of the future trend, at a small current cost. These are commonly called Options in the financial parlance. Such options give the right (not obligation) to buy or sell an asset in the future if the conditions become favourable. Myers (1977) identified that many corporate decisions can be viewed as exercising options on real assets, and hence coined the term real options. Options to delay, abandon or expand a project are all real options in the hand of a manager.

Real options, though based on the same basic philosophy as financial options, differ from the latter in certain distinct aspects. Real options span over real assets which often require time to build which widens the uncertainty. The time to expiration of real assets is much longer than that of financial options, and in some cases even have a perpetual existence. Most importantly, for a real option, the decision rule is more significant than the exercise value, while for a financial option it is the exercise value that assumes greater importance.

Real options provide an important tool for managers to capture value from uncertainty. Most of the corporate decisions are irreversible and hence a wrong decision can cause great losses for the company. However, if the manager uses the flexibility in decision making, by quantifying the value of this flexibility, he might be able to save costs. For example, capital budgeting is a critical decision a manager has to take. If the value of option to abandon or expand a project at later stage is captured, a not so attractive project might become viable. Similarly, while valuing a firm, traditional DCF methods might not be able to capture the options that might open up due to acquisition of the firm. Damodaran (2005) mentions that flexibilities offered by gain of control over a firm, option to liquidate and the option to utilize the patents, trademarks etc of the firm can be more effectively captured by real option valuation.

There has been a lot of research on real options and its applicability in various decision making scenarios. One of the areas of focus has been the development of a decision rule for sequential investment. Most of this research has concentrated on the mathematical formulation of the investment rule. This study attempts to develop an easily usable spreadsheet based framework, to enable a practicing manager to decide on such a rule. The study also attempts to capture multiple parameters that affect the project value of an oil company and thereby also calculate an average time to wait before making any investment.

It is observed, that the investment rule and the average time to wait depends not only on the price of oil, but it also is highly sensitive to the reserve value, recovery factor and exchange rate.

This paper is organized in five sections. In the second section we review relevant literature on real option. The third section discusses the unique characteristics of the oil and gas business. The fourth and fifth sections discuss the research methodology and the results respectively. The last section provides the concluding remarks.

Literature Review

Myers (1977), who coined the term Real Options, splits the value of a firm into what he calls "Real Assets" and "Real Options". He points out that a part of the value of a firm is made up of present value of options to make further investments and this value depends on the rule for deciding whether the options are to be exercised on possibly favourable terms. Myers says "One can think of real options that are separable, objectively identifiable, relatively long-lived, and for which reasonable secondary markets exist. Examples are patents, certain trademarks, franchises and operating licenses." Among other things, Myers applies the Real Options Theory to better understand the corporate borrowing decision.

Since then, Real Options Theory has been employed in specific industries, unlike Myers who considered a general firm. Banerjee (2003) looks at use to Real Options to better value a research-driven firm. He has particularly focused on a pharmaceutical firm and used Real Options to value the R&D investments which significantly improves the valuation as compared to a traditional DCF methodology.Real Options offer an effective methodology to value uncertainty in high-tech and R&D environments. Leon (2004) has used real options to value a biotech firm, while, Hartmann (2006) has used Real Options for valuation of pharmaceutical R&D. Similarly,Benaroch and Kauffman (1999) utilize option-pricing models to evaluation investment decisions in the Information Technology industry. They apply the Black Scholes option pricing model to a real world business simulation involving IT as its test bed.

Otherindustries which have seen significant applications of the Real Options theory are the oil and gas and mining industries. Gibson and Schwartz (1998) examine the oil price and convenience yield behaviours. Davis (1998) applies the real options theory to value a mineral reserve deposit. Abid and Kaffel (2009) use real option to evaluate the option of deferring an oilfield development. Major research has gone into exploring different ways of modelling the price risk and the geotechnical risk (Amstrong et al, 2004) which would be eventually used to value the oil field and the investment decision.

Oil and Gas Industry

Oil and gas business is one of the oldest and most important businesses of the modern world. Over the last century, the world has become excessively dependent on oil and many ancillary industries completely depend on oil and gas. In fact, Yergin(1991)argues thatoil is the reason for most major wars over the last century. The situation has become all the more critical over the past decade as easy oil is almost exhausted and most of the oil reserves are concentrated in the OPEC group of countries. All the recent discoveries are either not of significant size or located in very adverse conditions, raising the costs and uncertainties over oil production. Coupled with this, the dynamics and the power games between the OPEC and OECD blocks have caused oil price to be very volatile and subject to abrupt changes.

Oil and gas business possesses certain unique characteristics that distinguish it from other businesses. It is not only extremely capital intensive but also faces a vast plethora of uncertainties, besides the uncertainties of supply, demand and price. There are uncertainties with reserve volumes, recovery factors, oil quality, concessionary agreements and strategic interests. As such, the oil industry has not only learnt to live with uncertainty but also mastered the art of quantifying uncertainty and exploiting it.

However, the industry still widely uses the one-time period Bayesian uncertainty framework, which fails to recognize the multi-period decision making characteristics. One of the typical uncertainties facing an oil company is when to invest and when to withhold investments during the execution of an oil project such as oil exploration, oilfield development, well work-overs, logging, stimulation, fracking, water shut-offs etc. Each of these projects requires capital investment over a period of time, during which the managers have options to postpone, abandon or extend the project. This flexibility in making investment can prove to be great value as most of these investments are of immense magnitude and irreversible. One of the problems that the manager encounter in such cases is the difficulty is quantifying flexibilities to justify a seemingly unviable project. This can be achieved through the use of real options.

Methodology

Real options lend the managers a tool to not only deal with uncertainty but also exploit it. In a typical capital investment scenario, a manager has to decide when to invest for developing or exploring a particular asset. We attempt to first create a decision framework or investment envelope, which would mark the boundary of whether or not to invest.

For calculation of the investment envelope, the model used is the one used by Majid and Pindyck (1987) and as elaborated by Dixit and Pindyck (1993). Our hypothetical company invests continuously until project completion and has the option to stop the project at any point of time. The company can also restart the project at a later date with no extra cost. This is perhaps an over simplification but would serve to develop the model at this stage. The payoff upon completion, given by "V", follows an exogenous geometric brownian motion:

$$dV = \alpha V\,dt + \sigma V\,dz$$

It is assumed that the project value spans the range for V. Let "k" be the maximum rate of investment and "K" be the total remaining expenditure required to complete the project. Also, let μ be the risk adjusted discount rate applicable for V and let$\delta = \mu - \alpha$ (with $\alpha < \mu$), where α is the drift for the project value and δ is the convenience yield. Here investment is irreversible and if a firm suspends investment, because of V falling below a critical value, the firm can resume work at a later date from the same point. The rate of change of K is given by

$$dK = -I\,dt \qquad (1)$$

The solution of the model is a decision rule that would help the firm decide to continue or suspend investment at any point of time depending on the value of the project V. That is, to find out a critical cut-off for the project value, $V*(K)$, such that when $V \geq V*(K)$ the firm invests at rate "k", and does not invest otherwise. This creates an opportunity to postpone or an option for the firm which has a value of $F_v(V,K)$. As such, to develop the decision rule would be a solution of the equation for $F_v(V,K)$.

Let us consider a portfolio consisting of the option to invest and a short position in $F_V$units of V. The value of this portfolio is $\Omega = F(V,K)$ - $F_V\,V$. The change in the value of this portfolio is given by

$$d\Omega = dF - F_V\,dV$$

$$= F_V dV + 0.5F_{VV}(dV)^2 + F_k\,dK - F_V\,dV$$

$$= 0.5\sigma^2 V^2 F_{VV}\,dt - IF_k\,dt \qquad (2)$$

The short position would involve a cash outflow of $\delta\ F_V\ V\ dt$, with an additional outflow of$I\ dt$ for the investment. Therefore the total return on the portfolio is

$$d\Omega - Idt - \delta F_V V dt$$

For this portfolio to be risk free this must equal $r\ \Omega\ dt$, where r is the risk free rate. Substituting d Ω from above and dividing throughout by dt gives the equation for $F(V,K)$

$$0.5\sigma^2 V^2 F_{VV} - (r-\delta)VF_V - rF - IF_k - I = 0 \qquad (3)$$

The solution to the above equation should also satisfy the following boundary conditions

$$F(V,0) = V$$

$$F(0,K) = 0$$

$$\lim_{V\to\infty} F_v(V,K) = e^{-\delta K/k}$$

Along with the condition that $F(V,K)$ and $F_V(V,K)$should be continuous at $V = V*$.

Equation (3) is linear in I and hence the optimal investment rule is either to invest at maximum rate k or not invest at all. When there is no investment, i.e. $I = 0$, F_k term disappears and the equation reduces to an ordinary differential equation. This has an analytical solution (for $V<V*$), Dixit and Pindyck (1993).

$$F(V,K) = AV^{\beta 1} \qquad (4)$$

$$\beta 1 = 0.5 - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - 0.5\right)^2 + 2r/\sigma^2}$$

$A$is a constant which is obtained from the solution of $F(V,K)$for $V>V*$ and$I = k$.For these conditions equation (3) can be solved numerically (as elaborated in the Appendix).

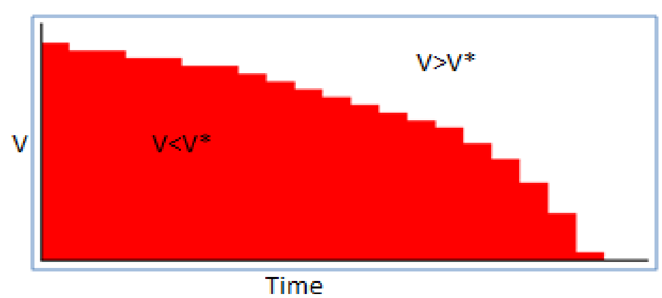

Figure 1: Investment Envelope

The investment envelope as calculated above would show regions wherein the investor should wait (red region) and regions wherein the investor should invest.

In most oil investment scenarios, it is not only important to find out whether there is an option value in postponing an investment, but it is also necessary to have an idea about what would be the average waiting time. This immensely helps in designing exploration and development strategies and securing rig and service contracts when the prices are appropriate. Though several researchers, like Dias &

Rocha (1999) and Dias (2002), have used the above framework for deciding on whether or not to invest, the time dimension implicit in the above model has not been properly exploited.

As the investment sequence used in the above model involves a constant rate of investment over a period of time and comparing the same to the project value to make a decision on investment, the time at which the decision rule gives a green light for investment is also the time when the project value attains a certain threshold. Hence the evolution of project value combined with the above decision rule can be used to estimate the average time to wait before making an investment.

Supposing that the current value of the project is such that the above model guides the investor to wait. In order to find out, on an average, how much time should he be waiting, we formulate a Monte Carlo framework.

Simulation is carried out to estimate an average time to wait, given an initial value for V, by simulating the evolution of V. Most of the work done on real options concentrates on the stochastic nature of the project value V or the associated price P. However, especially for an oil project the project value depends upon a number of variables which are themselves stochastic. Using a simplified form, V can be calculated as

$V = q * B * P * E$

Where$q$ = Recovery Factor

B = Reserve Size

P = Oil Price

E = Exchange Rate

For the above simulation to work, the factors comprising V should be independent. Recovery Factor is independent of the size of the field (and other parameters). Oil price and exchange rate do exhibit a certain degree of correlation for a majorly oil importing nation. However considering the plethora of other macroeconomic factors that affect the spot exchange rate, the effect of oil price is minimal and therefore the parameters can be taken as independent.

Recovery Factor ranges from 20% to about 70% [1]and depends on a number of geological factors such as type of deposit, primary and secondary drive mechanisms, fault connectivity etc. This is a stochastic variable but in the industry it is often expressed as a range typical for a type of oilfield. Reserve Size is again composed of a number of factors viz. porosity, area and height of deposit, oil formation volume factor and connate water saturation. Porosity is generally modelled as a random distribution while the other factors are modelled as ranges. Recovery factor and reserve size are handled as triangular distributions characterised by the parameters minimum, maximum and the mode. For the recovery factor, the minimum corresponds to the recovery for through drive for an oilfield whereas the minimum corresponds to the tertiary recovery using advanced techniques such as ASPs, microbial EOR etc. Mode corresponds to natural drive with water-flood and gas lift.Similarly the minimum, mode and maximum for the reserve size correspond to the P, 2P and 3P values of the reserve.

[1]Total E&P The Know-How series, 2008

Oil price and exchange rates are modelled as mean reverting geometric brownian motions as

$$\frac{dP}{P} = \varphi(\mu - P)dt + \sigma\, dz$$

Where φ= Speed of adjustment

μ= Mean price/exchange rate

This is a typical Arithmetic Ornstein-Uhlenbeckprocess where the half life (time taken for the expected value of the parameter to reach the mid -point between the current value and long run mean μ) is given by

$$H = \frac{\ln 2}{\varphi}$$

Oil price has been extensively modelled via geometric brownian motion, mean reverting models, jump diffusion models etc. Here oil price is modelled as a mean reverting Geometric Brownian Motion.Exchange rate assumes a lot of importance in the current global scenario, especially for majorly oil importing countries like India. As such, companies in these countries, while making investments abroad would need to take care of the volatility of the exchange rate. Exchange rate is also a random variable but the central bank usually intervenes to keep the exchange within a narrow range.

For each simulated time path (iteration) the simulator first samples the reserve size and recovery factor from their respective triangular distributions. Next for each time step, oil price and exchange rates are generated using the mean reverting geometric brownian models. Combining these parameters gives the value of the project V and thus its evolution over time.

Results

The framework obtainedprovides an easy decision tool to help a company decide to make an investment or wait and also provide the value for waiting.

Assuming the following typical parameters (Table 1) for a project envelope is constructed.

V maximum	Rs. 30,000 million
V Minimum	Rs. 400 million
Max Investment	Rs. 2,500 million
Max Investment Rate	Rs. 500 million/yr

Table 1: Parameters used for investment envelope calculation

The minimum and maximum values for the project should be sufficiently wide so as to span the range of possible project values to be explored in the simulation phase. Max investment corresponds to a 10 well development for a 4 million barrel oil reserve (assuming a per well investment of about 5 million USD).

The result for the investment envelope is shown in Fig 2.

Investment Remaining (million Rs)

Project Value (million Rs)	2500	2375	2250	2125	2000	1875	1750	1625	1500	1375	1250	1125	1000	875	750	625	500	375	250	125	0
50000	23815.19	24434.94	25072.7	25729.6	26406.97	27106.28	27829.25	28577.87	29354.47	30161.91	31003.41	31883.32	32807.21	33780.17	34812.15	35938.98	37039.29	38605.05	39858.34	40282.45	50000
44562.55	23921.9	24547.43	25191.5	25855.33	26540.32	27248.09	27980.5	28739.72	29528.36	30349.51	31207.02	32105.8	33051.56	34052.55	35125.47	36256.81	37560.24	38962.29	40218.27	44105.82	44562.55
39716.41	23525.35	24130.7	24752.81	25392.64	26051.3	26729.99	27430.12	28153.27	28901.29	29676.25	30480.79	31317.61	32190.69	33104.8	34061.39	35078.15	36166.22	37186.59	38879.21	39295.93	39716.41
35397.29	22662.27	23225.02	23800.95	24390.54	24994.3	25612.75	26246.41	26895.82	27561.41	28243.68	28942.7	29658.8	30390.98	31138.53	31896.69	32671.25	33388.82	34243.26	34624.45	35009.11	35397.29
31547.87	21409.72	21914.55	22428.14	22950.48	23481.44	24020.83	24568.32	25123.37	25685.28	26252.84	26824.69	27398.09	27970.64	28535.3	29091.84	29605.99	30129.97	30479.56	30832.39	31188.48	31547.87
28117.07	19870.84	20311.07	20755.92	21204.96	21657.67	22113.37	22571.2	23030.1	23488.66	23945.27	24397.52	24843.14	25277.45	25699.23	26094.5	26479.17	26800.66	27125.16	27452.71	27783.34	28117.07
25059.36	18157.49	18534.53	18913.07	19292.53	19672.24	20051.41	20429.13	20804.3	21175.72	21541.84	21901.3	22251.76	22592.69	22919.29	23237.63	23534.11	23833.4	24135.55	24440.57	24748.5	25059.36
22334.18	16374.19	16695.31	17016.14	17336.19	17654.89	17971.62	18285.73	18596.53	18903.25	19205.31	19501.86	19792.98	20077.49	20358.26	20632.47	20909.32	21188.84	21471.06	21756	22043.7	22334.18
19905.36	14606.76	14882.19	15156.77	15430.19	15702.13	15972.25	16240.27	16505.95	16769.14	17029.73	17288.05	17544.11	17799.43	18053.78	18310.62	18569.97	18831.86	19096.32	19363.37	19633.04	19905.36
17740.67	12917	13157.34	13397.1	13636.14	13874.37	14111.75	14348.25	14583.98	14819.07	15053.87	15288.67	15524.27	15760.91	15999.9	16241.27	16485.03	16731.21	16979.84	17230.94	17484.54	17740.67
15811.39	11342.96	11557.44	11771.94	11986.49	12201.14	12415.99	12631.19	12846.92	13063.46	13281.07	13500.19	13721.03	13944.09	14169.39	14396.96	14626.82	14859	15093.52	15330.41	15569.69	15811.39
14091.91	9902.896	10098.56	10294.89	10491.98	10689.93	10888.9	11089.03	11290.52	11493.56	11698.36	11905.07	12113.89	12324.84	12537.94	12753.21	12970.69	13190.39	13412.34	13636.56	13863.08	14091.91
12559.43	8600.797	8782.566	8965.485	9149.654	9335.182	9522.185	9710.789	9901.109	10093.27	10287.34	10483.42	10681.52	10881.68	11083.9	11288.23	11494.66	11703.24	11913.98	12126.91	12342.05	12559.43
11193.61	7431.792	7602.87	7775.389	7949.428	8125.066	8302.385	8481.454	8662.343	8845.098	9029.767	9216.37	9404.928	9595.463	9787.998	9982.553	10179.15	10377.82	10578.57	10781.44	10986.44	11193.61
9976.31	6386.405	6548.836	6712.839	6878.471	7045.782	7214.82	7385.626	7558.232	7732.668	7908.954	8087.112	8267.161	8449.123	8633.019	8818.872	9006.703	9196.534	9388.387	9582.286	9778.253	9976.31
8891.4	5453.36	5608.528	5765.263	5923.64	6083.693	6245.454	6408.946	6574.193	6741.214	6910.029	7080.659	7253.124	7427.446	7603.644	7781.74	7961.755	8143.712	8327.633	8513.538	8701.452	8891.4
7924.47	4621.162	4770.086	4920.575	5072.572	5226.201	5381.497	5538.479	5697.167	5857.579	6019.736	6183.658	6349.364	6516.875	6686.212	6857.395	7030.445	7205.385	7382.234	7561.016	7741.753	7924.47
7062.69	3878.088	4022.315	4167.376	4314.013	4461.958	4611.495	4762.675	4915.516	5070.039	5226.262	5384.204	5543.886	5705.328	5868.549	6033.571	6200.414	6369.099	6539.647	6712.08	6886.42	7062.69
6294.63	3232.665	3352.889	3495.12	3637.055	3780.738	3925.229	4071.238	4218.87	4368.143	4519.077	4671.69	4826.004	4982.036	5139.807	5299.338	5460.648	5623.759	5788.691	5955.465	6124.103	6294.63
5610.09	2694.659	2794.874	2913.434	3031.747	3171.268	3313.361	3454.995	3597.983	3742.577	3888.797	4036.662	4186.19	4337.401	4490.315	4644.952	4801.331	4959.473	5119.4	5281.131	5444.688	5610.09
5000	2246.192	2329.728	2428.556	2527.179	2643.48	2761.925	2905.141	3044.617	3185.042	3327.061	3470.692	3615.956	3762.87	3911.455	4061.729	4213.714	4367.429	4522.894	4680.13	4839.159	5000
4456.25	1872.362	1941.996	2024.376	2106.585	2203.53	2302.263	2421.643	2549.747	2688.137	2826.412	2966.271	3107.734	3250.818	3395.545	3541.932	3690	3839.768	3991.257	4144.488	4299.48	4456.25
3971.64	1560.749	1618.793	1687.463	1755.99	1836.801	1919.102	2018.614	2125.397	2240.756	2380.208	2516.705	2654.78	2794.452	2935.739	3078.662	3223.239	3369.49	3517.436	3667.096	3818.491	3971.64
3539.73	1300.996	1349.38	1406.622	1463.744	1531.106	1599.709	1682.66	1771.671	1867.831	1984.075	2116.03	2251.085	2387.715	2525.938	2665.772	2807.238	2950.355	3095.142	3241.62	3389.809	3539.73
3154.79	1084.474	1124.806	1172.52	1220.136	1276.287	1333.473	1402.618	1476.815	1556.971	1653.869	1763.863	1891.292	2025.21	2160.701	2297.784	2436.477	2576.799	2718.772	2862.414	3007.745	3154.79
2811.71	903.9869	937.6063	977.38	1017.071	1063.877	1111.545	1169.183	1231.032	1297.848	1378.619	1470.306	1576.528	1702.128	1835.184	1969.814	2106.035	2243.868	2383.332	2524.446	2667.231	2811.71
2505.94	753.5381	781.5623	814.7165	847.8018	886.8177	926.5529	974.598	1026.153	1081.849	1149.178	1225.606	1314.149	1418.846	1545.067	1677.51	1811.529	1947.143	2084.371	2223.232	2363.747	2505.94
2233.42	628.1282	651.4883	679.1248	706.7038	739.2263	772.3484	812.3975	855.3727	901.7991	957.9223	1021.631	1095.438	1182.71	1287.924	1416.994	1549.05	1682.686	1817.921	1954.775	2093.267	2233.42
1990.54	523.59	543.0624	566.0993	589.0884	616.1982	643.8079	677.1917	713.0146	751.7144	798.4971	851.6027	913.1261	985.8737	1073.577	1181.167	1315.116	1446.989	1580.448	1715.512	1852.202	1990.54
1774.07	436.4499	452.6815	471.8845	491.0475	513.6455	536.6602	564.4879	594.3489	626.608	665.6047	709.8721	761.1562	821.7966	894.9039	984.5873	1106.621	1236.923	1368.8	1502.269	1637.352	1774.07
1581.14	363.8124	377.3426	393.3496	409.3234	428.1605	447.3448	470.5413	495.4326	522.3228	554.8294	591.7294	634.4784	685.0265	745.9667	820.7243	922.4484	1049.702	1180.168	1312.216	1445.866	1581.14
1409.19	303.2638	314.5422	327.8852	341.2005	356.9025	372.8941	392.23	412.9787	435.3936	462.4902	493.249	528.8834	571.0189	621.8169	684.1327	768.9271	882.8415	1012.05	1142.831	1275.205	1409.19
1255.94	252.7921	262.1935	273.3159	284.4151	297.5039	310.834	326.9519	344.2474	362.9319	385.5188	411.1585	440.8623	475.9853	518.3291	570.2738	640.956	735.9119	862.2141	991.8665	1123.103	1255.94
1119.36	210.7204	218.5571	227.8284	237.0805	247.9909	259.1025	272.5379	286.9549	302.5298	321.3576	342.7302	367.4904	396.7679	432.0645	475.3642	534.2829	613.4355	728.6731	857.3191	987.5418	1119.36
997.63	175.6506	182.1831	189.9114	197.6236	206.7182	215.9806	227.1799	239.1976	252.1803	267.8747	285.6902	306.3297	330.7346	360.1569	396.2502	445.3632	511.3426	607.4014	737.4037	866.7228	997.63
889.14	146.4174	151.8627	158.3048	164.7335	172.3145	180.0353	189.3708	199.3884	210.2104	223.2928	238.1433	255.3478	275.6911	300.2166	330.303	371.2422	426.2408	506.3127	630.529	759.0427	889.14
792.45	122.0494	126.5885	131.9584	137.3172	143.6365	150.0724	157.8542	166.2045	175.2255	186.1306	198.5096	212.8508	229.8083	250.2521	275.3313	309.4571	355.3023	422.048	535.2768	663.0728	792.45
706.27	101.737	105.5206	109.9968	114.4638	119.7314	125.0961	131.5828	138.5434	146.0631	155.1532	165.472	177.4264	191.5618	208.6031	229.5084	257.9547	296.17	351.8074	450.3832	577.5395	706.27
629.46	84.80509	87.959	91.69026	95.41377	99.80471	104.2766	109.6837	115.4859	121.7541	129.3314	137.9328	147.8977	159.6805	173.8857	191.3118	215.0238	246.879	293.2567	375.4267	501.3078	629.46
561.01	70.69114	73.32015	76.43043	79.53424	83.1944	86.92205	91.42927	96.26581	101.4908	107.807	114.9769	123.2833	133.1052	144.9462	159.4721	179.2378	205.7914	244.4506	312.9452	433.3663	561.01
500	58.92614	61.11762	63.71025	66.2975	69.34851	72.45578	76.21287	80.24447	84.59984	89.86489	95.84153	102.7655	110.9527	120.8231	132.9315	149.4076	171.5419	203.7671	260.8623	206.8012	500

Figure 2: Generation of the Investment Envelope

The red zone in the investment envelope shows the zone wherein the company must not invest but wait. The numbers in the red zone show the option value for waiting. For example, when Rs. 2,000 million of investment is left, the company should not invest anymore if the project value falls below Rs. 6,294.63 million. At Rs. 5,610.09 million of project value, this value of option for waiting is worth Rs. 3,031.75 million.

Though this model has been developed for oil investments, this investment envelope is a generic framework and can be used by any firm to decide on its investments.

To find out the average time that the company needs to wait before making the investment, a Monte Carlo simulation is run with the following parameters (Table 2):

	Oil Price ($/bbl)	Currency (Rs/$)
Mean	80	47
Speed of Adjustment	0.1	0.6
Current	90	45
	Reserve Size (MMbbl)	Recovery Factor (%)
Min	1	40
Max	4	70
Mode	2	60

Table 2: Simulation run parameters

Mean oil price is obtained from the EIA forecast[1] while volatility corresponds to the OVX index (oil Volatility Index, CBOE). Speed of adjustment for oil corresponds to a half-life of about 1.1 years, Dias & Rocha (1999). Exchange rate used is the USD to INR exchange. Speed of adjustment for exchange corresponds to a half-life of six months and volatility is obtained from standard industry data[3].

[1]http://www.eia.gov/forecasts/steo/uncertainty/index.cfm
[2]http://www.cboe.com/micro/oilvix/introduction.aspx
[3]http://www.ratesfx.com/predictions/pred-inr.html

The simulator also draws the project value path on the envelope and marks the point where the project value breaches the envelope, as shown in figure 3. On running the simulator for a number of runs, the average time to wait is obtained. For example, on 100 simulation runs, the average time to wait comes out to be 1.035yr.

Investment Remaining (million Rs)

Project Value (million Rs)	2500	2375	2250	2125	2000	1875	1750	1625	1500	1375	1250	1125	1000	875	750	625	500	375	250	125	0
50000	23815.19	24434.94	25072.7	25729.6	26406.97	27106.28	27829.25	28577.87	29354.47	30161.91	31003.41	31883.32	32807.21	33780.17	34812.15	35938.98	37039.29	38605.05	39858.34	40282.45	50000
44562.55	23921.9	24547.43	25191.5	25855.33	26540.32	27248.09	27980.5	28739.72	29528.36	30349.51	31207.02	32105.8	33051.56	34052.55	35125.47	36256.81	37560.24	38962.29	40218.27	44105.82	44562.55
39716.41	23525.35	24130.7	24752.81	25392.64	26051.3	26729.99	27430.12	28153.27	28901.29	29676.25	30480.79	31317.61	32190.69	33104.8	34061.39	35078.15	36166.22	37186.59	38879.21	39295.93	39716.41
35397.29	22662.27	23225.02	23800.95	24390.54	24994.3	25612.75	26246.41	26895.82	27561.41	28243.68	28942.7	29658.8	30390.98	31138.53	31896.69	32671.25	33388.82	34243.26	34624.45	35009.11	35397.29
31547.87	21409.72	21914.55	22428.14	22950.48	23481.44	24020.83	24568.32	25123.37	25685.28	26252.84	26824.69	27398.09	27970.64	28535.3	29091.84	29605.99	30129.97	30479.56	30832.39	31188.48	31547.87
28117.07	19870.84	20311.07	20755.92	21204.96	21657.67	22113.37	22571.2	23030.1	23488.66	23945.27	24397.52	24843.14	25277.45	25699.23	26094.5	26479.17	26800.66	27125.16	27452.71	27783.34	28117.07
25059.36	18157.49	18534.53	18913.07	19292.53	19672.24	20051.41	20429.13	20804.3	21175.72	21541.84	21901.3	22251.76	22592.69	22919.29	23237.63	23534.11	23833.4	24135.55	24440.57	24748.5	25059.36
22334.18	16374.19	16695.31	17016.14	17336.19	17654.89	17971.62	18285.73	18596.53	18903.25	19205.31	19501.86	19792.98	20077.49	20358.26	20632.47	20909.32	21188.84	21471.06	21756	22043.7	22334.18
19905.36	14606.76	14882.19	15156.77	15430.19	15702.13	15972.25	16240.27	16505.95	16769.14	17029.73	17288.05	17544.11	17799.43	18053.78	18310.62	18569.97	18831.86	19096.32	19363.37	19633.04	19905.36
17740.67	12917	13157.34	13397.1	13636.14	13874.37	14111.75	14348.25	14583.98	14819.07	15053.87	15288.67	15524.27	15760.91	15999.9	16241.27	16485.03	16731.21	16979.84	17230.94	17484.54	17740.67
15811.39	11342.96	11557.44	11771.94	11986.49	12201.14	12415.99	12631.19	12846.92	13063.46	13281.07	13500.19	13721.03	13944.09	14169.39	14396.96	14626.82	14859	15093.52	15330.41	15569.69	15811.39
14091.91	9902.896	10098.56	10294.89	10491.98	10689.93	10888.9	11089.03	11290.52	11493.56	11698.36	11905.07	12113.89	12324.84	12537.94	12753.21	12970.69	13190.39	13412.34	13636.56	13863.08	14091.91
12559.43	8600.797	8782.566	8965.485	9149.654	9335.182	9522.185	9710.789	9901.109	10093.27	10287.34	10483.42	10681.52	10881.68	11083.9	11288.23	11494.66	11703.24	11913.98	12126.91	12342.05	12559.43
11193.61	7431.792	7602.87	7775.389	7949.428	8125.066	8302.385	8481.454	8662.343	8845.098	9029.767	9216.37	9404.928	9595.463	9787.998	9982.553	10179.15	10377.82	10578.57	10781.44	10986.44	11193.61
9976.31	6386.405	6548.836	6712.839	6878.471	7045.782	7214.82	7385.626	7558.232	7732.668	7908.954	8087.112	8267.161	8449.123	8633.019	8818.872	9006.703	9196.534	9388.387	9582.286	9778.253	9976.31
8891.4	5453.36	5608.528	5765.263	5923.64	6083.693	6245.454	6408.946	6574.193	6741.214	6910.029	7080.659	7253.124	7427.446	7603.644	7781.74	7961.755	8143.712	8327.633	8513.538	8701.452	8891.4
7924.47	4621.162	4770.086	4920.575	5072.572	5226.201	5381.497	5538.479	5697.167	5857.579	6019.736	6183.658	6349.364	6516.875	6686.212	6857.395	7030.445	7205.385	7382.234	7561.016	7741.753	7924.47
7062.69	3878.088	4022.315	4167.376	4314.013	4461.958	4611.495	4762.675	4915.516	5070.039	5226.262	5384.204	5543.886	5705.328	5868.549	6033.571	6200.414	6369.099	6539.647	6712.08	6886.42	7062.69
6294.63	3232.665	3352.889	3495.12	3637.055	3780.738	3925.229	4071.238	4218.87	4368.143	4519.077	4671.69	4826.004	4982.036	5139.807	5299.338	5460.648	5623.759	5788.691	5955.465	6124.103	6294.63
5610.09	2694.659	2794.874	2913.434	3031.747	3171.268	3313.361	3454.995	3597.983	3742.577	3888.797	4036.662	4186.19	4337.401	4490.315	4644.952	4801.331	4959.473	5119.4	5281.131	5444.688	5610.09
5000	2246.192	2329.728	2428.556	2527.179	2643.48	2761.925	2905.141	3044.617	3185.042	3327.061	3470.692	3615.956	3762.87	3911.455	4061.729	4213.714	4367.429	4522.894	4680.13	4839.159	5000
4456.25	1872.362	1941.996	2024.376	2106.585	2203.53	2302.263	2421.643	2549.747	2688.137	2826.412	2966.271	3107.734	3250.818	3395.545	3541.932	3690	3839.768	3991.257	4144.488	4299.48	4456.25
3971.64	1560.749	1618.793	1687.463	1755.99	1836.801	1919.102	2018.614	2125.397	2240.756	2380.208	2516.705	2654.78	2794.452	2935.739	3078.662	3223.239	3369.49	3517.436	3667.096	3818.491	3971.64
3539.73	1300.996	1349.38	1406.622	1463.744	1531.106	1599.709	1682.66	1771.671	1867.831	1984.075	2116.03	2251.085	2387.715	2525.938	2665.772	2807.238	2950.355	3095.142	3241.62	3389.809	3539.73
3154.79	1084.474	1124.806	1172.52	1220.136	1276.287	1333.473	1402.618	1476.815	1556.971	1653.869	1763.863	1891.292	2025.21	2160.701	2297.784	2436.477	2576.799	2718.772	2862.414	3007.745	3154.79
2811.71	903.9869	937.6063	977.38	1017.071	1063.877	1111.545	1169.183	1231.032	1297.848	1378.619	1470.306	1576.528	1702.128	1835.184	1969.814	2106.035	2243.868	2383.332	2524.446	2667.231	2811.71
2505.94	753.5381	781.5623	814.7165	847.8018	886.8177	926.5529	974.598	1026.153	1081.849	1149.178	1225.606	1314.149	1418.846	1545.067	1677.51	1811.529	1947.143	2084.371	2223.232	2363.747	2505.94
2233.42	628.1282	651.4883	679.1248	706.7038	739.2263	772.3484	812.3975	855.3727	901.7991	957.9223	1021.631	1095.438	1182.71	1287.924	1416.994	1549.05	1682.686	1817.921	1954.775	2093.267	2233.42
1990.54	523.59	543.0624	566.0993	589.0884	616.1982	643.8079	677.1917	713.0146	751.7144	798.4971	851.6027	913.1261	985.8737	1073.577	1181.167	1315.116	1446.989	1580.448	1715.512	1852.202	1990.54
1774.07	436.4499	452.6815	471.8845	491.0475	513.6455	536.6602	564.4879	594.3489	626.608	665.6047	709.8721	761.1562	821.7966	894.9039	984.5873	1106.621	1236.923	1368.8	1502.269	1637.352	1774.07
1581.14	363.8124	377.3426	393.3496	409.3234	428.1605	447.3448	470.5413	495.4326	522.3228	554.8294	591.7294	634.4784	685.0265	745.9667	820.7243	922.4484	1049.702	1180.168	1312.216	1445.866	1581.14
1409.19	303.2638	314.5422	327.8852	341.2005	356.9025	372.8941	392.23	412.9787	435.3936	462.4902	493.249	528.8834	571.0189	621.8169	684.1327	768.9271	882.8415	1012.05	1142.831	1275.205	1409.19
1255.94	252.7921	262.1935	273.3159	284.4151	297.5039	310.834	326.9519	344.2474	362.9319	385.5188	411.1585	440.8623	475.9853	518.3291	570.2738	640.956	735.9119	862.2141	991.8665	1123.103	1255.94
1119.36	210.7204	218.5571	227.8284	237.0805	247.9909	259.1025	272.5379	286.9549	302.5298	321.3576	342.7302	367.4904	396.7679	432.0645	475.3642	534.2829	613.4355	728.6731	857.3191	987.5418	1119.36
997.63	175.6506	182.1831	189.9114	197.6236	206.7182	215.9806	227.1799	239.1976	252.1803	267.8747	285.6902	306.3297	330.7346	360.1569	396.2502	445.3632	511.3426	607.4014	737.4037	866.7228	997.63
889.14	146.4174	151.8627	158.3048	164.7335	172.3145	180.0353	189.3708	199.3884	210.2104	223.2928	238.1433	255.3478	275.6911	300.2166	330.303	371.2422	426.2408	506.3127	630.529	759.0427	889.14
792.45	122.0494	126.5885	131.9584	137.3172	143.6365	150.0724	157.8542	166.2045	175.2255	186.1306	198.5096	212.8508	229.8083	250.2521	275.3313	309.4571	355.3023	422.048	535.2768	663.0728	792.45
706.27	101.737	105.5206	109.9968	114.4638	119.7314	125.0961	131.5828	138.5434	146.0631	155.1532	165.472	177.4264	191.5618	208.6031	229.5084	257.9547	296.17	351.8074	450.3832	577.5395	706.27
629.46	84.80509	87.959	91.69026	95.41377	99.80471	104.2766	109.6837	115.4859	121.7541	129.3314	137.9328	147.8977	159.6805	173.8857	191.3118	215.0238	246.879	293.2567	375.4267	501.3078	629.46
561.01	70.69114	73.32015	76.43043	79.53424	83.1944	86.92205	91.42927	96.26581	101.4908	107.807	114.9769	123.2833	133.1052	144.9462	159.4721	179.2378	205.7914	244.4506	312.9452	433.3663	561.01
500	58.92614	61.11762	63.71025	66.2975	69.34851	72.45578	76.21287	80.24447	84.59984	89.86489	95.84153	102.7655	110.9527	120.8231	132.9315	149.4076	171.5419	203.7671	260.8623	206.8012	500
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Time(yrs)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75	5
$/bbl	101	100.9359	100.819	100.5305	100.3734	100.3492	100.2664	100.19	100.0486	99.63527	99.8432	99.62987	99.69845	99.75927	99.86132	99.6535	99.52807	99.42111	99.18639	99.31555	99.00064
Rs/$	50	48.96111	48.22628	47.80454	47.48552	47.35452	47.25882	47.16993	47.11326	47.08098	47.05767	47.04352	47.06615	47.05981	47.07064	47.00872	47.00515	47.04435	46.99438	47.04738	47.07495

Figure 3: Simulation run showing project value evolution

Conclusions

The current paper develops a framework for application of the Majid and Pindyck (1987) model in oil investment scenarios. The paper also expands the scope by exploiting the time dimension and using a Monte Carlo framework to estimate an average time to wait.The model framework developed can be used as a tool to decide on whether or not to invest in the current market scenario. A manager can plug in the values for maximum investment, maximum rate of investment, risk free rate etc and generate an investment envelope which would help him to decide whether to invest now or to postpone his investment. The model would also help him plan his course of action by calculating on an average how much should be the waiting time for the next investment. Though developed for an oil and gas investment, the same model can be used for a general project by using the appropriate parameters. The three uncertain parameters used in the simulation can be replaced with such other parameters directly affecting the project value.

One of the major drawbacks of the previous model is that while project value evolves with time, the project costs (rate of investment "I") is assumed to be constant. This cost uncertainty can be captured as well because this also opens up the opportunity to delay an investment decision based on the value versus cost equation. Also the investment schedule is assumed to be quarterly. This assumption can be relaxed.

Other things that can be incorporated are different models for price and exchange rate. Price and exchange rate futures rates from the market can also be incorporated to base the investment decision on the current market sentiment about the parameters. In fact, the model can be programmed to extract such date in real time from databases such as Bloomberg or Reuters and the same can be used to make a decision.

The reserve size used can also be more explicitly modelled. Normally, exploratory logs give a pretty good idea about the distributions of porosity and connate water saturation, while laboratory experiments can give an idea about the oil formation volume factor. Moreover, seismic data can provide depths to about +/- 40 ft accuracy. The areal extent is however more tricky. Combining these parameters can give a better picture about the uncertainty range in the reserve value.

The current model as such can be enhanced to include the aforesaid ideas and can be a scope for further research.

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Appendix

In order to calculate the investment envelope, equation (3) must be solved. This is a PDE with no closed form analytical solution. As such, the equation needs to be solved numerically.

Equation (3) can be solved by Finite Difference Method (FDM) as mentioned by Dixit and Pindyck (1993). We reproduce the same below.

F(V,K) can first be transformed to G(X,K) by

$$F(V,K) = e^{-\frac{rK}{k}} G(X,K)$$

where X= log V.

The partial equation for V > V* and the boundary conditions then become

$$0.5\sigma^2 G_{XX} + (r - \delta - 0.5\sigma^2)G_X - kG_k - ke^{\frac{rK}{k}} = 0 \quad (5)$$

$$G(X,0) = e^X \quad (6)$$

$$\lim_{X\to\infty}(e^{-X}\, e^{-\frac{rK}{k}} G_X(X,K) = e^{-\delta K/k} \quad (7)$$

$$G(X^*,K) = \frac{1}{\beta 1} * G_X(X^*,K) \quad (8)$$

Using FDM method transforms the continuous variables V and K into discrete variables and replaces the partial derivatives with finite differences.

Let $G(X,K) \equiv G(i\Delta X, j\Delta K) \equiv G_{i,j}$, where -b ≤ I ≤ m and 0 ≤ j ≤ n. Substituting

$$G_{XX} \approx (G_{i+1,j} - 2G_{i,j} + G_{i-1,j})/\Delta X^2$$

$$G_X \approx (G_{i+1,j} - G_{i-1,j})/(2\Delta X)$$

$$G_K \approx (G_{i,j+1} - G_{i,j})/\Delta K$$

into equation (5)

$$G_{i,j} = p^+ G_{i+1,j-1} + p^0 G_{i,j-1} + p^- G_{i-1,j-1} - n_{j-1} \quad (9)$$

where,

$$p^+ = \Delta K(\frac{\sigma^2}{\Delta X} + r - \delta - 0.5\sigma^2)/(2k\Delta K)$$

$$p^0 = 1 - \sigma^2 \Delta K/(k\Delta X^2)$$

$$p^- = \Delta K(\frac{\sigma^2}{\Delta X} - r + \delta + 0.5\sigma^2)/(2k\Delta K)$$

$$n_j = \Delta K e^{rj\Delta K/k}$$

The terminal boundary condition is

$$G_{i,j=0} = e^{i\Delta X} \tag{10}$$

and the upper boundary condition is

$$\lim_{X\to\infty}(e^{-X}e^{-\frac{rK}{k}}G_X(X,K) = e^{-\delta K/k}$$

or

$$G_X(m\Delta X, K) = e^{m\Delta X(r-\delta)j\Delta K/k}$$

using FDM approximation for G_X, the above equation becomes

$$\frac{G_{m+1,j} - G_{m-1,j}}{2\Delta X} = e^{m\Delta X(r-\delta)j\Delta K/k}$$

or

$$G_{m+1,j} = 2\Delta X e^{m\Delta X(r-\delta)j\Delta K/k} + G_{m-1,j}$$

substituting this for G_{m+1}, in equation (9) (setting i=m)

$$G_{m,j+1} = p^+G_{m+1,j} + p^0G_{m,j} + p^-G_{m-1,j} - n_j$$

or

$$G_{m,j+1} = p^+2\Delta X e^{m\Delta X(r-\delta)j\Delta K/k} + p^0G_{m,j} + (p^- + p^+)G_{m-1,j} - n_j \tag{11}$$

The free boundary condition is

$$G_{i^*,j} = G_{i^*+1,j}/(\beta 1\Delta X + 1)$$

The solution proceeds in the following manner

1. First the terminal boundary is filled in using equation (10)

2. Then for j = 1 to n, equation (11) is used to calculate $G_{m,j}$ and then using (9) rest of the $G_{i,j}$ down the column for I < m is calculated.

3. For each value of $G_{i,j}$, the following check is performed to find if the boundary has been reached

$$G_{i^*,j} - G_{i^*+1,j}/(\beta 1\Delta X + 1) \leq \varepsilon,$$

where ε is chosen arbitrarily

4. At the free boundary, value of A as in equation (4) is calculated and the same equation is used to fill the values of $G_{i,j}$ in the lower region.

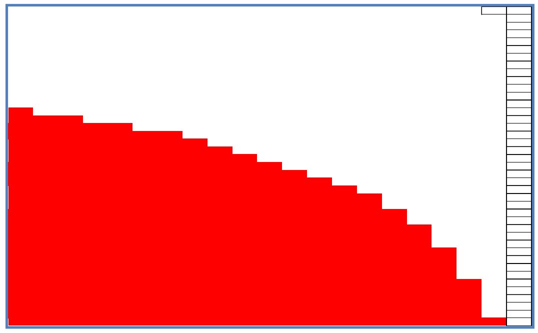

Figure 4: Sequence of FDM computation

As shown in Figure 4, the computation starts from the extreme right most top cell and then continues down the column. As the G values hit the envelope the analytical solutions of equation (4) are used to calculate the value of the option down the column.